Motorcycle Speed Estimates Using Conservation of Linear and Rotational Momentum

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Abstract

This paper discusses the conflicting published information on the use of conservation of linear momentum in motorcycle/automobile collisions. The proper methodology for both linear and angular momentum analyses in motorcycle collisions is reviewed and two case studies are included as examples of successful use of these techniques. The use of linear and angular momentum in collisions where significant weight disparities exist between the vehicles should always include a sensitivity analysis that evaluates the level of confidence of the speed estimates. Use of the sensitivity analysis will allow the reconstructionist to determine if the techniques should be applied to the given analysis or be abandoned in favor of other methods of speed analysis.

Background

For many years there has been some controversy over the use of conservation of linear momentum to estimate the speed of motorcycles involved in collisions with other motor vehicles. Fricke and Riley indicate in Topic 874 of the Traffic Accident Investigation Manual that ***"occasionally a momentum analysis is attempted"*** and that this technique ***"rarely… works well"*** in accurately estimating the speed of the motorcycle. They go on to explain that the heading and departure angles become sensitive ***"when the angles of approach are nearly collinear and the weight difference between the colliding vehicles is fairly large."*** [1]

In 1990, Brown and Obenski write that a momentum analysis ***"can sometimes be used in motorcycle accidents,"*** and give a graphical example of a momentum vector diagram of a motorcycle/automobile collision.[2] In 1994, Obenski further clarifies this position by stating ***"Generally it is tricky to use momentum analysis in accidents between vehicles with a big weight difference,"*** but gives the same graphical example as in his previous work. Obenski specifically cautions against using a momentum analysis where the automobile has been moved very little after impact with the motorcycle.[3]

In 1990, Niederer wrote about techniques that may be used to reconstruct motorcycle/vehicle collisions, with the emphasis of the paper on the use of conservation of linear and angular momentum. Niederer specifically cautions that ***"due to the often unfavourable mass ratio an accurate reconstruction may be impeded,"*** but concludes that when used cautiously, the use of momentum and other available information ***"represents a powerful tool for motorcycle-vehicle collision reconstruction."*** He further concludes that the reconstructionist should assess the sensitivity of the momentum analysis to changes in variation of impact configuration and post-impact trajectory. [4]

Conservation of Linear Momentum

The ***Law of Conservation of Momentum*** dictates that the total momentum just prior to two vehicles colliding is the same as the total momentum just after the collision.

Equation 1				
Explanation	A typical mathematical representation of two passenger vehicles that collide.			
Formula	$M_1\vec{V}_1 + M_2\vec{V}_2 = M_1\vec{V}_3 + M_2\vec{V}_4$			
Where	**M_1**	Mass of vehicle 1	**V_1**	Velocity of vehicle 1 at impact
	M_2	Mass of vehicle 2	**V_2**	Velocity of vehicle 2 at impact
			V_3	Velocity of vehicle 1 after impact
			V_4	Velocity of vehicle 2 after impact

In most motorcycle collisions this basic formula must be expanded to include both motorcycle and rider post-impact trajectory, since the motorcycle and rider seldom stay together following the collision.

Equation 2				
Explanation	A typical representation of a motorcycle/vehicle impact, where the motorcycle and rider have different post-impact trajectories.			
Formula	$M_1\vec{V}_1 + (M_2 + M_3)\vec{V}_2 = M_1\vec{V}_3 + M_2\vec{V}_4 + M_3\vec{V}_5$			
Where	M_1	Mass of vehicle 1	V_1	Velocity of vehicle 1 at impact
	M_2	Mass of motorcycle	V_2	Velocity of motorcycle/rider at impact
	M_3	Mass of rider	V_3	Velocity of vehicle 1 after impact
			V_4	Velocity of motorcycle after impact
			V_5	Velocity of rider after impact

Since momentum is a vector quantity, **Equations 1 and 2** account for both the speed of the objects and the direction of travel. The following formulae can be used to solve for the speed of vehicle 1 and vehicle 2 when the initial direction of travel of vehicle 1 is determined to be zero degrees.

Equation 3		
Note	Equations 3 and 4 must be solved in order, since Equation 4 requires the value of V_2 from Equation 3 for solution.	
Formula	$V_2 = \frac{M_1V_3Sin\theta + M_2V_4Sin\phi + M_3V_5Sin\gamma}{(M_2 + M_3)Sin\psi}$	
Where	ψ	Approach angle of motorcycle
	ϕ	Departure angle of motorcycle
	γ	Departure angle of rider
	θ	Departure angle of vehicle 1

Equation 4		
Formula	$V_1 = \frac{M_2V_4Cos\phi + M_3V_5Cos\gamma - (M_2 + M_3)V_2Cos\psi}{M_1} + V_3Cos\theta$	
Where	ψ	Approach angle of motorcycle
	ϕ	Departure angle of motorcycle
	γ	Departure angle of rider
	θ	Departure angle of vehicle 1

Example I

A Suzuki motorcycle was traveling east on Route 66 when it collided with the right side of a BMW that was traveling south. Prior to striking the BMW, the motorcycle skidded for a distance of approximately 80 feet, leaving a single skid mark in the approximate center of the eastbound travel lane (Photo 1). The collision produced significant damage to the motorcycle and the automobile, with the motorcycle puncturing the right side of the vehicle and entering the rear seat area of the occupant compartment. The BMW rotated in a clockwise direction and rolled onto its roof while traveling to its final rest position. Near the final rest position, the A-pillar and roof line of the BMW made contact with the curbing and the vehicle came to rest in contact with the curb. The Suzuki and its rider remained within the BMW and came to final rest at the same location as the BMW (Photo 2).

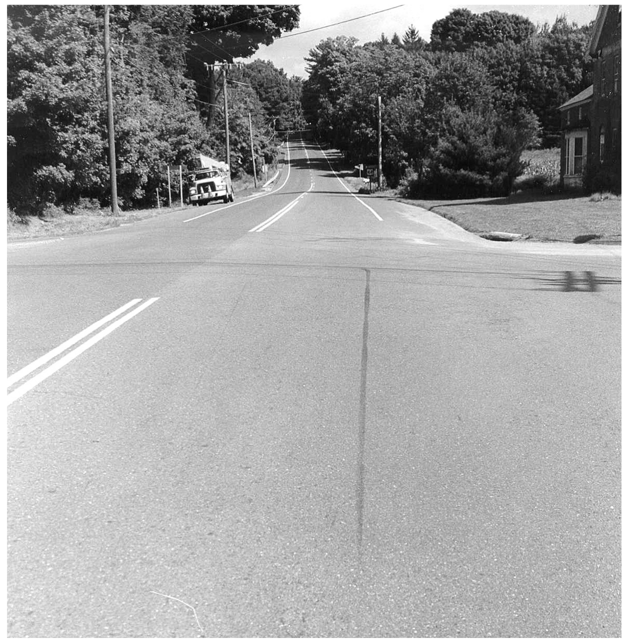


Photo 1 – Accident scene showing tire marks

Photo 2 – Post impact positions of car and motorcycle

The police documented the physical evidence on the roadway, including the skid mark left by the motorcycle, gouge marks near the point of impact and the final rest positions of the motorcycle and the BMW. Using a total station survey instrument, measurements of the collision locus were gathered and a scale diagram was created. The police measurements were placed onto the scale diagram and the vehicles were placed into their estimated impact positions (Figure 1).

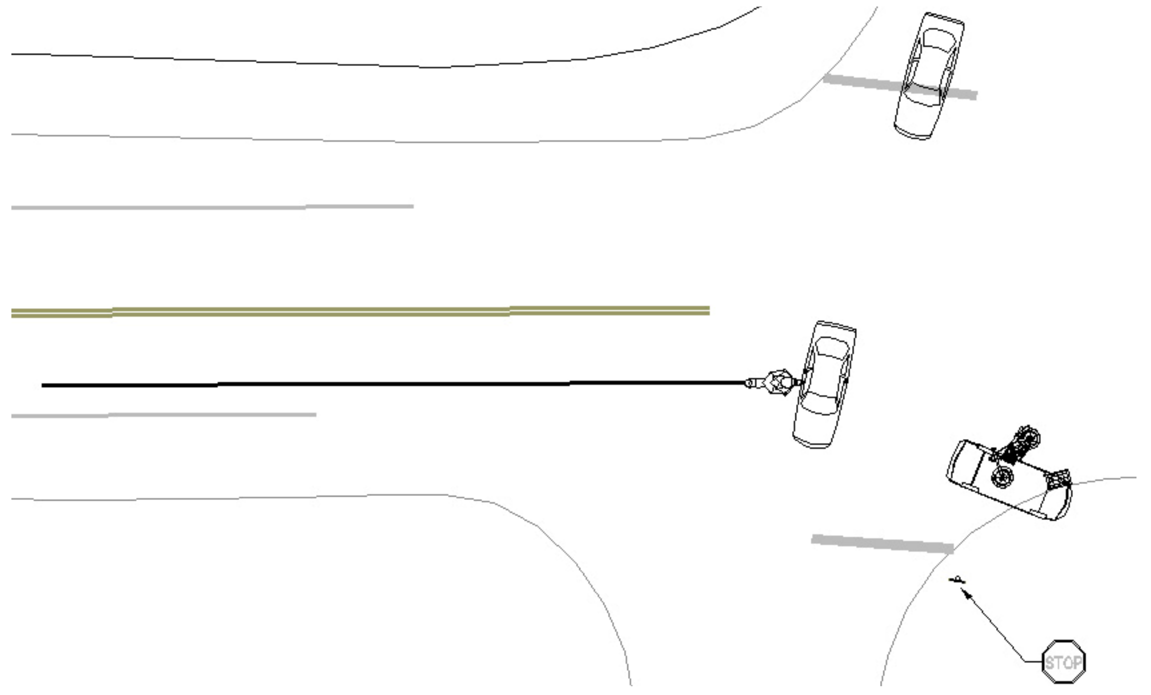


Figure 1 – Scale diagram of Example I

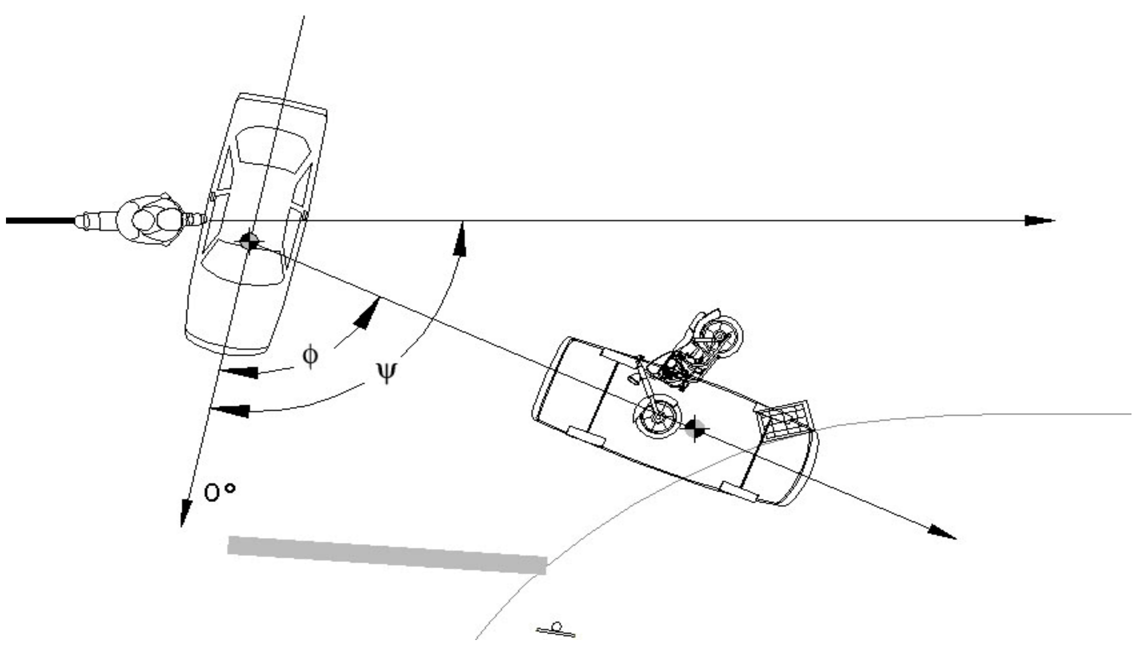


Figure 2 - Scale Diagram of Example I 1

From the scale diagram information we measured the approach and departure angles necessary for a momentum calculation (Figure 2). The post-impact distance traveled by the BMW while rolling over and sliding on its roof was used to estimate the post-impact speed of the vehicle. Since the BMW, Suzuki and the rider traveled to final rest together, the same post-impact speed was used for all three.

The following values were used in a momentum calculation for this collision.

Values used for Example I					
V_3	20 MPH	Speed of BMW after impact	θ	75 deg.	Departure angle of BMW/Suzuki
	0 deg.	Angle of BMW at impact	M_1	2800 lbs.	Weight of BMW and driver
ψ	100 deg.	Angle of Suzuki at impact	M_2	755 lbs.	Weight of Suzuki and rider

For purposes of this analysis, we can derive specific formulae that evaluate two pre-impact units that travel to final rest as one unit. The following formulae are representative of this type of trajectory.

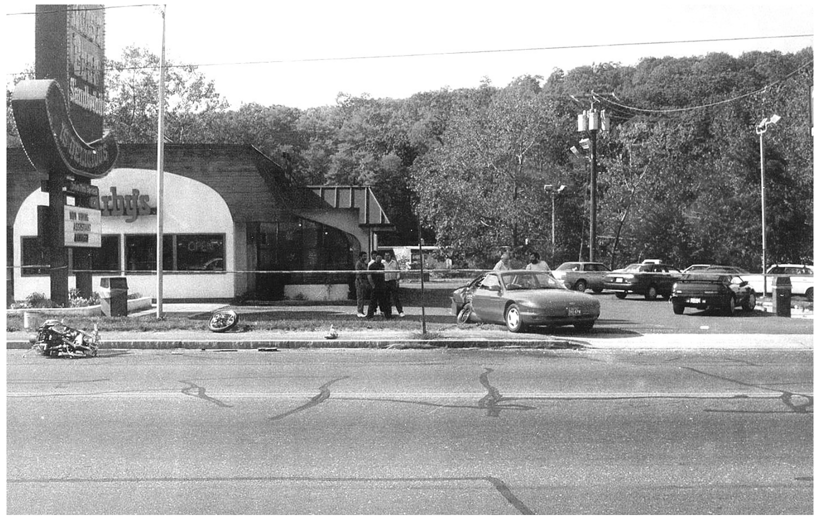
The analysis which follows indicates that the motorcycle was traveling at a speed of approximately 92 miles per hour when it struck the BMW, which was traveling at a speed of approximately 11 miles per hour. These speeds are a good starting point in our analysis and these speed estimates will be evaluated later in the uncertainty analysis section of this paper.

Solving for V_2 in Example I	
Step 1	$V_2 = \dfrac{(M_1 + M_2)V_3 Sin\theta}{M_2 Sin\psi}$
Step 2	$V_2 = \dfrac{(2800 + 755)(20)Sin(75)}{(755)Sin(100)}$
Solution	$V_2 = 92.4$ mph

Solving for V_1 in Example I	
Step 1	$V_1 = \dfrac{(M_1 + M_2)V_3 Cos\theta - M_2 V_2 Cos\psi}{M_1}$
Step 2	$V_1 = \dfrac{(3555)(20)Cos(75) - (755)(92.4)Cos(100)}{2800}$
Solution	$V_1 = 10.9$ mph

Example II

This collision occurred when the driver of an eastbound Ford Probe made a left turn across two lanes of traffic toward a restaurant parking lot entrance. The driver of the Ford reportedly started to make her turn from a stopped position and did not see the westbound Harley Davidson motorcycle until she was already well into her turn.



Photos 3/4 – Final rest positions of car and motorcycle

The motorcycle collided with the right rear wheel area of the Ford, ejecting the rider and causing significant damage to both vehicles. The rider, who had significant interaction with the right C-pillar area of the Ford, was thrown for an overall distance of approximately 81 feet. The Ford was rotated a total of approximately 145 degrees, traveling over a 7-inch barrier curb and coming to final rest in the restaurant entrance (Photos 3/4). The motorcycle sustained severe front fork deformation and slid on its side to final rest in the roadway.

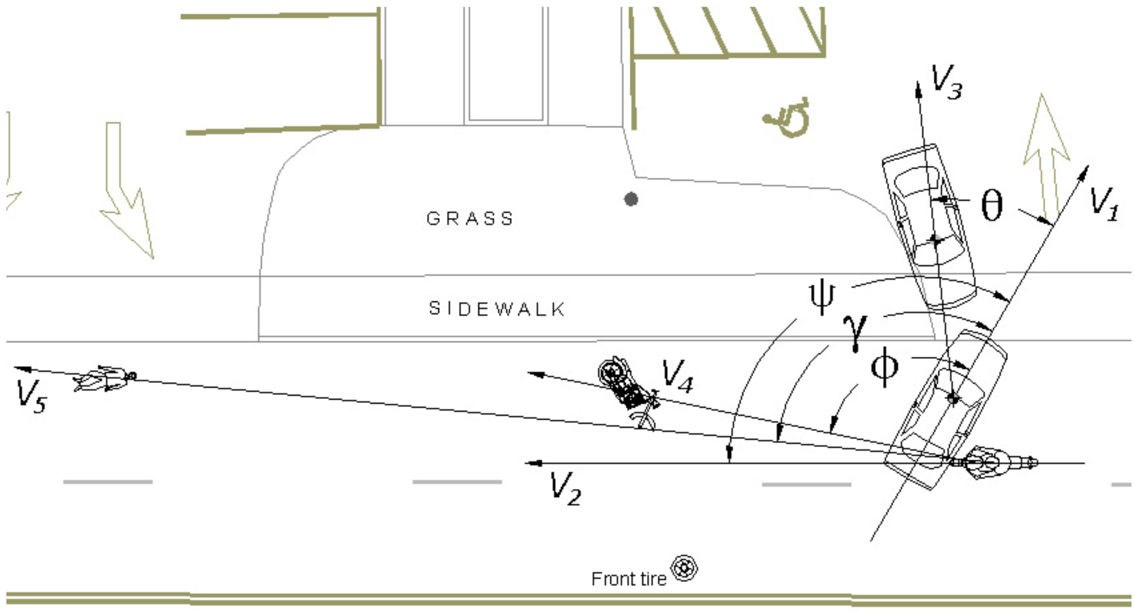


Figure 3 – Scale diagram of Example II

The following values were used to perform a momentum calculation to estimate the speed of both involved vehicles.

Values used for Example II					
	0 deg.	Approach angle of Ford	V_3	12 MPH	Departure speed of Ford
ψ	120 deg.	Approach angle of motorcycle	V_4	19 MPH	Departure speed of motorcycle
θ	36 deg.	Departure angle of Ford	V_5	38 MPH	Departure speed of rider
ϕ	108 deg.	Departure angle of motorcycle	M_1	2770 lbs.	Weight of Ford
γ	115 deg.	Departure angle of rider	M_2	613 lbs.	Weight of motorcycle
			M_3	206 lbs.	Weight of rider

Using **Equation 3** we can first solve for the motorcycle speed.

Solving for V_2 in Example II	
Step 1	$V_2 = \dfrac{M_1 V_3 Sin\theta + M_2 V_4 Sin\phi + M_3 V_5 Sin\gamma}{(M_2 + M_3) Sin\psi}$
Step 2	$V_2 = \dfrac{19538 + 11077 + 7094.6}{709.3}$
Solution	$V_2 = 53.2 mph$

Substituting the value of V_2 from the above calculation allows us to calculate the value of V_1 with **Equation 4.**

Solving for V_1 in Example II	
Step 1	$V_1 = \frac{M_2V_4Cos\phi + M_3V_5Cos\gamma - (M_2 + M_3)V_2Cos\psi}{M_1} + V_3Cos\theta$
Step 2	$V_1 = \frac{(613)(19)Cos(108) + (206)(38)Cos(115) - (613+206)(53.2)Cos(120)}{2770} + (12)Cos(36)$
Step 3	$V_1 = \frac{(-3599.1) + (-3308.3) - (-21785.4)}{2770} + 9.7$
Solution	$V_1 = 15.1 mph$

The calculations indicate that the motorcycle was traveling at a speed of approximately 53 miles per hour at the moment it made contact with the Ford, which was traveling at approximately 15 miles per hour. We will evaluate the confidence level of these speed estimates in the uncertainty analysis section.

Speed Estimates from Vehicle Rotation

Many times it is possible to evaluate the rotation of the automobile created by the motorcycle impact to estimate the speed of the motorcycle at impact. A large number of motorcycle collisions occur when the passenger vehicle crosses the path of the motorcycle while making a left turn or while crossing an intersection.[5] This creates an angle between the motorcycle and the passenger vehicle and many times produces an eccentric impact on the automobile, which will tend to rotate the vehicle.

In those collisions where proper documentation of the vehicle rotation has occurred, speed calculations based on this rotation can be completed. The accuracy of the calculations, as with all reconstruction calculations, is dependent upon the quality of information available to the reconstructionist. In modern police investigations, an increasing number of serious collisions are being documented with the use of total station survey instruments. This methodology allows for much more accurate placement of scene evidence on scale diagrams or collision maps, which are typically used by the reconstructionist to measure angles and distances during the analysis.

This section will describe the use of rotational mechanics to evaluate a specific type of impact. For a more thorough description of rotational mechanics, we suggest one review Chapter 15 of **Fundamental of Applied Physics for Traffic Accident Investigators** by Daily and Shigemura.[6]

The first step in the rotational analysis is to determine the total amount of torque acting through the tires/roadway interaction to slow the angular velocity of the vehicle following the collision. In many motorcycle/passenger vehicle collisions, the impulse applied to the vehicle results in the struck end of the vehicle "sliding," while the opposite end acts a pivot point. When this type of vehicular motion occurs, it is prudent to calculate the torque acting on the vehicle in the following manner.

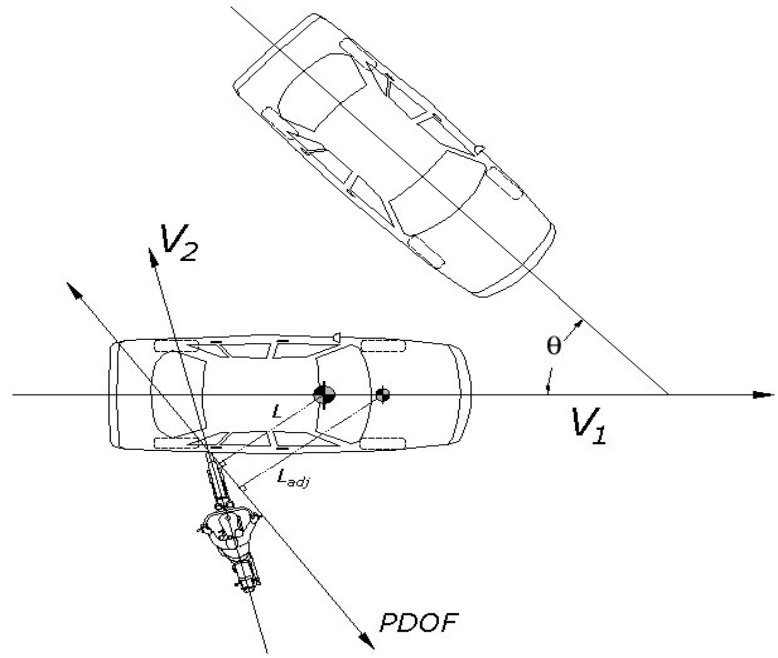
Equation 5				
Explanation	Calculates the amount of torque acting on the vehicle.			
Formula	$\tau_{tire} = WB \cdot W_a \cdot f$			
Where	τ_{tire}	Torque caused by tires sliding sideways	W_a	Weight on axle closest to damage centroid
	WB	Wheelbase of vehicle	**f**	Coefficient of friction of roadway

The value of torque calculated in **Equation 5** can then be used in the following formula, which calculates the rotational velocity of the vehicle.

Equation 6				
Explanation	Incorporates the parallel axis method to determine the moment of inertia for the vehicle while pivoting on one axle instead of rotating about its center of mass.			
Formula	$\omega = \sqrt{\dfrac{2 \cdot \tau_{tire} \cdot \theta}{I + M_1 \cdot D_{com}^{\ 2}}}$			
Where	τ_{tire}	Torque acting through tires	M_1	Mass of vehicle
	ω	Rotational velocity of vehicle in radians/second	**I**	Yaw moment of inertia of vehicle
	θ	Angle of rotation of the vehicle in radians	D_{com}	Distance of the farthest axle from the contact to the center of mass
Note	One can convert angular displacement in degrees by dividing the number by 57.3, since 2π radians = 360 degrees.			

The variables in **Equation 6** include value for moment of inertia for the automobile, which can be reasonably estimated by using the methods described by Garrot and by MacInnis, et al.[7,8]

Figure 4 - Angles of Impact and Rotation 1



After determining the angular velocity of the vehicle due to the impulse, we can calculate the change in velocity experienced by the motorcycle in the event. This is accomplished through the following formula.

Equation 7				
Explanation	Calculates the change in velocity experienced by the motorcycle in the event.			
Formula	$\Delta V_m = \frac{(I + M_1 \cdot D_{com}{}^2) \cdot \omega}{L \cdot M_m}$			
Where	**ΔV_m**	Change in velocity of motorcycle	**M_1**	Mass of vehicle
	ω	Angular velocity of vehicle	**I**	Yaw moment of inertia of vehicle
	L	Length of moment arm (PDOF to center of front axle)	**D_{com}**	Distance of the farthest axle from the contact to the center of mass

Since the above formula is calculating the change in velocity for the motorcycle in the collision, if we know the direction of travel of the motorcycle at impact and its post-impact velocity, we can calculate the impact speed of the motorcycle. The following formula can be used to calculate the initial speed of the motorcycle. A derivation for **Equation 8** can be found in **Appendix B**.

Equation 8				
Explanation	Calculates the initial speed of the motorcycle.			
Formula	$V_2 = V_4 \cdot Cos(\psi - \phi) + \sqrt{\Delta V_m^2 - (V_4 \cdot Sin(\psi - \phi))^2}$			
Where	V_2	Initial speed of the motorcycle	ϕ	Departure angle of motorcycle
	V_4	Post-impact speed of the motorcycle	ψ	Approach angle of motorcycle

Example III

From **Example II** of the Linear Momentum section of this paper, we find the following values:

Values used for Example III					
θ	145 deg or 2.53 rad.	Angular displacement of automobile	**L**	8.5 feet	Moment arm
I	1617.3 lb/ft/sec^2	Yaw moment of inertia for automobile	D_{com}	3.2 feet	Distance from front axle to center of mass
W_t	2770 lbs.	Total weight of automobile	**f**	.75 g	Roadway coefficient of friction
W_f	1745 lbs.	Weight on front axle	ϕ	108 deg.	Departure angle for motorcycle
W_r	1025 lbs.	Weight on rear axle	ψ	120 deg.	Approach angle for motorcycle
WB	8.6 feet	Wheelbase	M_m	21.2 slugs	Mass of motorcycle

Using the above-described methodology, we can estimate the speed of the motorcycle at the time of impact through the rotation experienced by the automobile with which it collided. First, we can calculate the amount of torque acting on the vehicle as it rotates to final rest using **Equation 5**.

Solving for τ_{tire} in Example III	
Step 1	$\tau_{tire} = WB \cdot W_a \cdot f$
Step 2	$\tau_{tire} = (8.6)(1025)(.75)$
Solution	$\tau_{tire} = 6611.25 lb \cdot ft$

Next, we determine the post-impact angular velocity of the vehicle using **Equation 6.**

Solving for T in Example III	
Step 1	$\omega = \sqrt{\dfrac{2 \cdot \tau_{tire} \cdot \theta}{I + M_1 \cdot D_{com}^2}}$
Step 2	$\omega = \sqrt{\dfrac{(2)(6611.25)(2.53)}{1617.3 + (86.02)(10.24)}}$
Solution	$\omega = 3.66 rad / \sec$

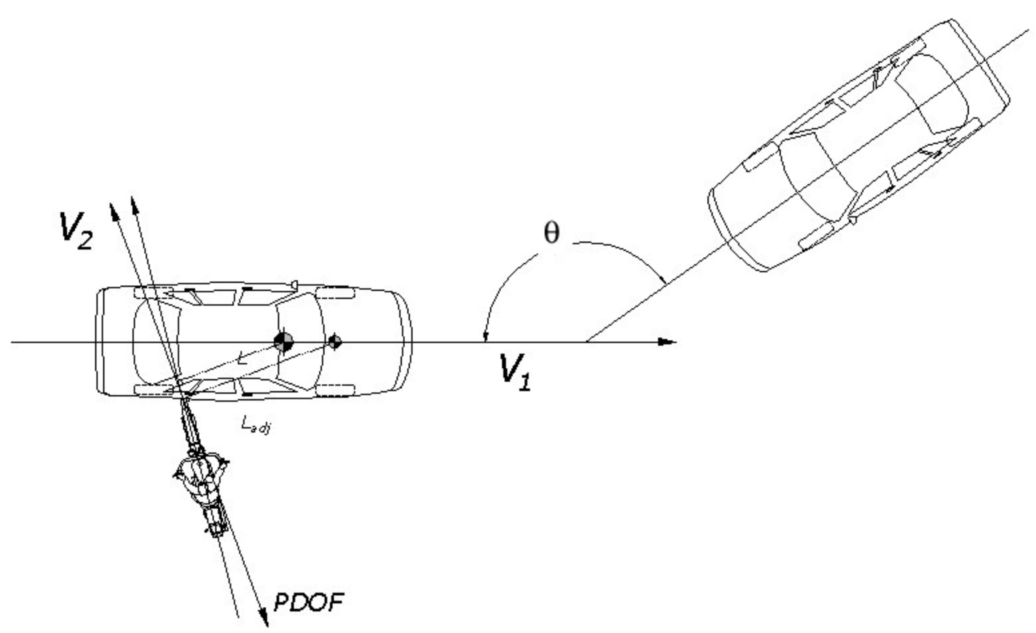


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Figure 5 – Angles of Rotation in Example III

Using **Equation 7**, we can calculate the change in velocity experienced by the motorcycle. It should be noted that the mass of the motorcycle includes approximately one-third the mass of the rider, since the rider was ejected from the collision with only partial interaction with the vehicle.

Solving for)V_m in Example III	
Step 1	$\Delta V_m = \dfrac{(I + M_1 \cdot D_{com}^2)\omega}{L \cdot M_m}$
Step 2	$\Delta V_m = \dfrac{(1617.3 + 86.02 \cdot 10.24)3.66}{8.5(21.2)}$
Solution	$\Delta V_m = 50.7\ ft/\sec$
or	$\Delta V_m = 34.6 mph$

The final step in the methodology is to use **Equation 8** to estimate the initial speed of the motorcycle.

Solving for V_2 in Example III	
Step 1	$V_2 = V_4 Cos(\psi - \phi) + \sqrt{\Delta V_m^2 - (V_4 Sin(\psi - \phi))^2}$
Step 2	$V_2 = (19)(.9781) + \sqrt{1197.2 - ((19)(.2079))^2}$
Solution	$V_2 = 52.9 mph$

Through the described methodology, we have calculated the speed of the motorcycle to be approximately 53 miles per hour. This speed estimate is consistent with that found by performing a linear momentum calculation using the same collision data. Because the two methods are similar but independent means of calculating the speed of the motorcycle, they complement one another when performing a collision analysis of this nature. Although one method alone may be used to estimate the speed of a motorcycle involved in a collision, because of the sensitivity of the momentum calculations, the application of both methods in conjunction with one another reduces the possibility of erroneous results.

There are times, however, that it is not possible to use both methods in conjunction with one another. A good example of this is **Example I** in this paper, where the BMW both rotated and rolled following impact. With the given information, one cannot reasonably determine the amount of rotation that occurred while the vehicle was airborne, so the above-described methodology cannot be used to estimate the resultant angular velocity of the vehicle due to the collision. When this is the case, it is especially important to determine the sensitivity of the calculations to reasonable changes in values.

Uncertainty and Sensitivity

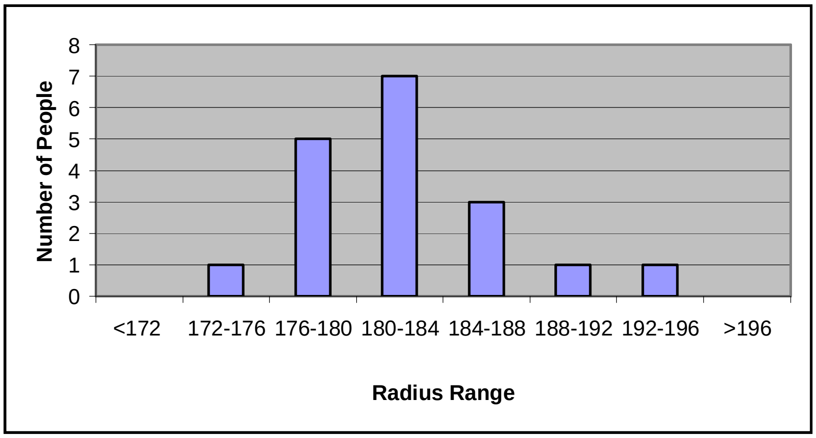
In the examples we included in this paper, we have used the physical evidence available at the scene and on the vehicles to estimate the initial speed of the motorcycle. This scene evidence was well documented and could be reasonably corroborated by photographs taken of the evidence. However, regardless of how carefully distances, angles, drag factors, grades, slopes, etc. are measured at a scene, there is always a range of uncertainty to every measurement.[9,10] Approach and departure angles can usually be determined from scene diagrams to within a narrow range, but the exact value can never be determined. These combined sources of uncertainty affect the level of confidence in our resultant calculations.

To evaluate the uncertainty of an overall analysis, there are several tools available, including Monte Carlo analysis, partial differentiation of the equations involved, and a numerical approach to the partial differentiation method. The first two methods are beyond the scope of this article, but the curious reader is directed to almost any recent statistical textbook, as well as papers by Tubergen and Kost and Werner.[11,12] The last method, a numerical approach to the partial differentiation, makes use of spreadsheets in an extension of the model outlined by Metz and Metz in 1998.[13]

Finding the absolute maximum and minimum values is a fairly simple, but sometimes tedious exercise, since we must perform calculations using the high/low combinations of all variables. Given that there are two choices for each variable of a function, the highest and lowest value, there will be 2^n possible permutations. If you have two variables (skid to stop has distance and drag factor, for instance) there will be ($2^2 = 4$) four possible permutations. With three variables there are ($2^3 = 8$) eight permutations, etc. One can see that with a complex momentum function, such as in Equations 3 and 4 where the total number of combinations is ($2^{10} = 1024$) 1024 calculations, this process would be extremely time consuming absent a spreadsheet program. This process will yield one highest possible value, one lowest possible value, and a number of intermediate values that really have no meaning or use in the reconstruction process.

The chance that all the values will combine in just the right way to allow one of these absolute highest or lowest values to occur is fleetingly small, so we must narrow the range of probability. This is where statistics starts to come into play. It has been found that any time the same quantity is measured repeatedly the results vary, if just a little, from measurement to measurement. As an example, 18 people attending last year's Special Problems Conference were given tape measures and asked to measure the chord and middle ordinate of an arc chalked onto the pavement.[9] After converting the participants' chord/middle ordinate values to radius values, the data looked like this:

Measurement Results

Radius Range (feet)	Number of People Reporting a Value in that Range	Histogram
<172	**0**	
172-176	**1**	
176-180	**5**	
180-184	**7**	
184-188	**3**	
188-192	**1**	
192-196	**1**	
>196	**0**	

(Measuring a chalked constant-radius arc. ($n = 18, \overline{X} = 182.3\, ft., s = 4.5\, ft.$)

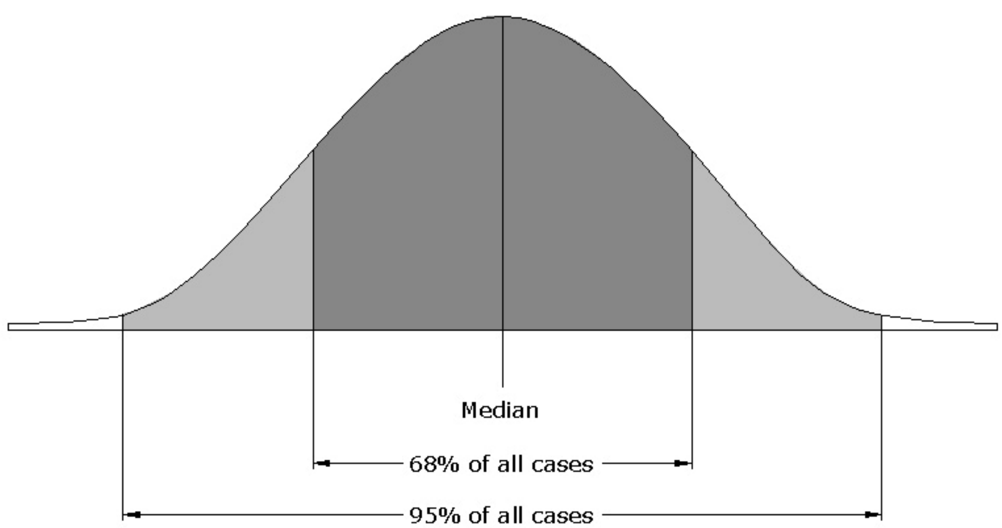


Figure 6 – Distribution Graph

Assuming the data followed a normal distribution (which is true for many measurements), the average value of all results was 182.3 feet, with a standard deviation of 4.5 feet. That means that 68.3% of all data fell within 4.5 feet of the average value. A normal distribution looks like Figure 6. The area bounded by one standard deviation around the mean is the darker shaded area, and represents about 68% of the total area under the curve.

If we spread our area of interest one more standard deviation, or twice the value of one standard deviation, in each direction, we will have shaded 95% of the total area. This is represented by the lighter shaded area seen in Figure 6. So getting back to our example, with a mean of 182.3 feet and a standard deviation of 4.5 feet, if we had had 100 participants, we would expect 68 of them to report a radius of between 177.8 and 186.8 (average plus/minus one sigma). We would expect that 95 people would report values between 173.3 and 191.3 feet (average plus/minus two sigma). Thus we can be 95% confident that the true value is between 173.3 feet and 191.3 feet.

If we can assign a 95% confidence level to each of the variables in an accident analysis, we can evaluate the range of the result to the same confidence level through this procedure:

Step 1	Determine the average (or nominal) and 95% confidence values for each variable.
Step 2	Calculate the nominal result using all the nominal values.
Step 3	One at a time, with all other values set at the nominal value, set each variable to its highest value and calculate the difference between the new value and the nominal value.
Step 4	Square these new values and add them together, then take the square root of the sum. This process is called a Root-Sum-of-Squares.[14]
Step 5	Now repeat the process for all the lowest values.
Step 6	Take the average of the two resulting values. This represents the range above and below the nominal value within which we can be 95% confident the true value lies.

Example

If you want to calculate the skid to stop distance with perception reaction time included, you can use this formula:

Skid-to-Stop Distance with Perception/Reaction Time				
Formula	$d = v \cdot t + \dfrac{v^2}{2 \cdot f \cdot g}$			
Where	**d**	Distance (feet)	**t**	Perception/Response time (sec)
	g	Gravity (ft/sec/sec)	**f**	Effective drag factor (g's)
	v	Initial velocity (ft/sec)		

Using these inputs and 95% confidence intervals:

v = 45 ± 8 ft/sec (one sigma=4 ft/sec)
f = 0.75 ± 0.12 g's (one sigma = 0.06 g's)
t = 1.5 ± 0.4 sec (one sigma = 0.2 sec)

We can run through the 8 permutations of high/low values to find the absolute highest and absolute lowest value values to be 65 to 170 feet. The chances of getting all three variables lined up to allow one of these values to happen is very slim indeed. To narrow the range, we first find the nominal value:

Solving for the nominal value	
Formula	$d_{nom} = (45)(1.5) + \frac{45^2}{(2)(0.75)(32.2)} = 109$

Then cycle each value to the high end of its range, and find the difference from the nominal value:

Solving for the high end values	
	$HI_{diff\,1} = \left((53)(1.5) + \frac{53^2}{(2)(0.75)(32.2)} \right) - d_{nom} = 28.7$
	$HI_{diff\,2} = \left((45)(1.5) + \frac{45^2}{(2)(0.87)(32.2)} \right) - d_{nom} = -5.4$
	$HI_{diff\,3} = \left((45)(1.9) + \frac{45^2}{(2)(0.75)(32.2)} \right) - d_{nom} = 18.4$
Take the square root of the sum of the squares	$HI_{diff} = \sqrt{(28.7)^2 + (-5.4)^2 + (18.4)^2} = 34.5$

Solving for the low end values	
	$LO_{diff1} = \left((37(1.5) + \dfrac{37^2}{(2)(0.75)(32.2)} \right) - d_{nom} = -25.2$
	$LO_{diff2} = \left((45)(1.5) + \dfrac{45^2}{(2)(0.63)(32.2)} \right) - d_{nom} = 8.4$
	$LO_{diff3} = \left((45)(1.1) + \dfrac{45^2}{(2)(.75)(32.2)} \right) - d_{nom} = -17.6$
Take the square root of the sum of the squares	$LO_{diff} = \sqrt{(-26)^2 + (-19.5)^2 + (-17.624.4)^2} = 32.3$

By taking the average LO_{diff} and HI_{diff} values, we find the 95% confidence range to be ± 33.2 feet. So, with 95% confidence we can say that given the starting values and ranges, the vehicle could have stopped in 109 ± 33 feet, or 76 to 142 feet.

Clearly, if we can narrow the ranges for the inputs, we will narrow the result range as well. If we are willing to live with the range that's only 68% likely (one standard deviation), then we can simply take half of the range found to be 95% likely. Using the previous example, we can say with 68% confidence that the vehicle could have stopped in 109 ± 16.6 feet, or about 92 to 126 feet. It is worth noting that both of these ranges are narrower than the absolute high and low of 65 to 170 feet found using our original 95% confidence limits. This is why evaluating the uncertainty can help us narrow the range of our final answers.

A separate but also important concept in any accident reconstruction is that of sensitivity. Rather than describing how tight a range the final answer can be reported, sensitivity analysis gives a means of evaluating how important each variable in the analysis is to the final answer. For example, if while measuring a critical speed yaw the chord is reported to be 50 feet, while the middle ordinate is reported to be 20 inches (yielding a nominal radius of 188.3 feet), a variation of 1 foot in the chord value changes the calculated radius by 7.4 feet, while changing the middle ordinate by one foot changes the calculated radius by 280.8 feet. Clearly this analysis is much more sensitive to the middle ordinate measurement.

As one goes through the earlier noted steps to conduct a numerical uncertainty analysis, each step provided us with the chance to see how much changing each variable changed the outcome. The distance to stop calculation was clearly least sensitive to the initial speed of the vehicle and most sensitive to the reaction time of the driver. Knowing which parameters have the greatest affect on the outcome can assist us in determining where to spend our energy to make particularly careful measurements. For instance, getting the middle ordinate right is a LOT more important than getting the chord right.

By using a spreadsheet to evaluate the linear and rotational momentum calculations, one may easily evaluate the potential sensitivity of speed estimates by changing the values of a single variable or combinations of changes to several variables. If, for example, one is reasonably sure that the angle of approach for the motorcycle is within ± 3 degrees, one can perform calculations at the limits of that range to evaluate the sensitivity of the approach angle to the resultant speed calculation. The same can be performed with the reasonable ranges of all of the variables that are contained within the particular formula used to estimate the speed of the motorcycle.

We encourage the investigator to adapt these described methods to fit the circumstances of the particular collision they are evaluating. For examples of the methods of uncertainty analysis described in this section performed on the previously described examples in **Conservation of Linear Momentum** and **Speed Estimates from Vehicle Rotation** sections of this paper, please refer to **Appendix C, D and E**. These examples are in a spreadsheet format, which displays the appropriate high and low calculation values for each of the variables and the 68% and 95% confidence ranges.

Other Methods of Estimating Speed

Searle's Method

A popular method used in the accident reconstruction community in motorcycle collision analysis is to estimate the speed of the motorcycle at the moment of impact by evaluating the total distance the rider was thrown from the point of impact to point of final rest. Searle developed the most commonly used formula in this type of analysis, which he described in his 1983 and 1993 papers.[15,16] Unfortunately, there is significant misapplication and misunderstanding of the Searle method, even though the technical papers he presented on the subject are reasonably clear.

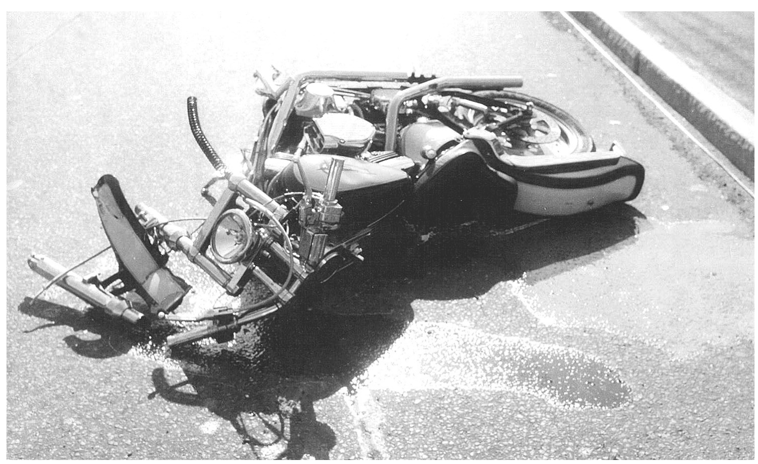


Photo 5 – Damage to motorcycle handlebars and tank due to rider interaction.

Although the Searle method purports to estimate the speed of the motorcycle at the moment of impact, what it is actually accomplishing is something slightly different. The formula is calculating the speed of the rider following his separation from both the motorcycle and the struck vehicle. In nearly every motorcycle collision where the rider is vaulted from the motorcycle, there is evidence of interaction between the rider, the motorcycle and the struck vehicle. Since this interaction is never addressed by the Searle formula, any resultant speed calculation using this formula must be a conservative value.

Searle warned users of his formula of the problem of rider/vehicle interaction in his technical paper, however, he does not adequately forewarn of the potential underprediction of actual motorcycle speed in nearly all angular impacts between motorcycles and automobiles. In **Example II**, we calculated the speed of the motorcycle to be approximately 38 miles per hour using the Searle formula for the overall throw distance of the rider. Using both rotational and linear momentum methods in this example, we find that the probable speed of the motorcycle was approximately 53 miles per hour. The difference between the speed estimates is, in our opinion, due solely to interaction between the motorcycle rider, the motorcycle and the struck vehicle. There was evidence on the motorcycle that the rider contacted the tank with his legs and folded the handlebars forward at impact (Photo 5). There was also damage to the upper C-pillar of the Ford due to interaction with the rider. This type of interaction would cause the rider to lose enough of his kinetic energy to account for the disparity in speed estimates.

Although the Searle method is a valid methodology to estimate motorcycle speeds, it will only reasonably estimate the speed of the motorcycle in limited circumstances. At times there will be so limited an amount of information available to the investigator that the only method available to perform a speed estimate is the Searle method. The investigator should thoroughly understand that the results of the speed analysis in these circumstances are most likely conservative estimates of speed for the motorcycle.

Speed from Motorcycle Fork Deformation

Another method of estimating motorcycle impact speed is to compare the amount of front fork deformation to that seen in crash tests of motorcycles. Speed estimates from front fork deformation originated with the work of Severy in 1970.[17] In this paper, Severy reported the amount of front fork deformation that resulted in crash tests performed with motorcycles.

These motorcycles were almost all of relatively small size and mass when compared to more modern motorcycles, and because of the significant changes in the design of motorcycles since Severy's crash tests, the use of this data on modern crash reconstructions is questionable. However, there are other sources of data on fork deformation that are more current and, therefore, more applicable to modern motorcycle collisions.[18,19,20] These more modern tests may be helpful in reasonably estimating the rate at which the front forks of newer motorcycles deform.

One obvious limitation to this methodology is the lack of specific data on a large variety of motorcycles. When performing a crush analysis on a vehicular crash, one can generally locate frontal crush data on nearly all production automobiles by checking the NHTSA database. There is no similar database for motorcycles, and the investigator is typically forced to compare front fork deformation to similar-sized motorcycles.

Another limitation to this methodology is that the front forks on motorcycles have a limited possible range of rearward deflection. After the front forks have been displaced rearward so that there is significant contact with the engine and frame of the motorcycle, the rate of deformation changes drastically because of the stiffer structures of the frame and engine being loaded. Under these conditions, an accurate estimate of the speed change of the motorcycle is not possible. It is only possible under these circumstances to estimate the minimum change in velocity experienced by the motorcycle. With more modern motorcycles, many of which are equipped with aluminum wheels, a fracture of the front wheel may occur if it strikes a hard spot on the other vehicle, making it difficult to estimate the energy absorbed by the motorcycle in the impact.

Provided there is adequate information available on the motorcycle, and the previously discussed limitations have not been exceeded, estimating speed change from front fork deflection is a potential tool for the accident reconstructionist. This method is most useful when the impact speed is relatively low, since the front forks of the motorcycle will typically be forced rearward into the frame and engine at speeds of more than about 35 miles per hour.

Discussion

The two examples we used for this paper both involve a motorcycle that weighs significantly less than the vehicle with which it collided, but was traveling much faster than the automobile at the moment of impact. By using a sensitivity analysis on hypothetical collisions involving vehicles of various weight disparity and impact speeds, one can start to understand why the historical literature on this subject sometimes discouraged the use of linear momentum to determine the speed of a motorcycle. These treatises almost always give examples of a motorcycle colliding with a vehicle that weighs much more than the motorcycle and the resultant automobile motion is relatively minor.

In these circumstances, one can understand that momentum techniques are probably not appropriate, since even small changes in the input variables will produce large changes in the calculated speed of the motorcycle. For example, a motorcycle that weighs 450 pounds, with a rider that weighs 175 pounds, strikes the side of a stationary sport-utility vehicle that weighs 4,200 pounds. The impact causes the center of mass of the vehicle to move sideways a distance of approximately 1 foot on a roadway surface with a measured coefficient of friction of approximately .75. Using a simple minimum speed formula, we find that the post-impact speed of the sport-utility vehicle was approximately 4.7 miles per hour.

Since the motorcycle collided with the side of the other vehicle, the high profile of the sport-utility vehicle produced a barrier that essentially prevented the motorcyclist from being thrown free. For calculation purposes we will assume that the motorcycle, the rider and the sport utility vehicle all attained a common post-impact velocity. Since the motorcycle and rider were providing all of the pre-impact momentum, we can use the following formula to estimate the speed of the motorcycle at the moment of impact.

Equation 9				
Explanation	Calculates the initial speed of the motorcycle.			
Formula	$(M_m + M_r) \cdot V_1 = (M_{veh} + M_m + M_r) \cdot V_2$			
or	$V_1 = \dfrac{(M_{veh} + M_m + M_r) \cdot V_2}{M_m + M_r}$			
Where	V_1	Initial speed of the motorcycle	M_m	Mass of motorcycle
	V_2	Post-impact speed of the motorcycle/vehicle	M_r	Mass of rider
			M_{veh}	Mass of vehicle

Solving for V_1 using Equation 9	
Step 1	$V_1 = \dfrac{(M_{veh} + M_m + M_r) \cdot V_2}{M_m + M_r}$
Step 2	$V_1 = \dfrac{(4200 + 450 + 175) \cdot 4.7}{450 + 175}$
Step 3	$V_1 = \dfrac{22677.5}{625}$
Solution	$V_1 = 36.3 mph$

Using a single variable sensitivity analysis on only the post-impact distance used to calculate the post-impact speed of the involved vehicles, we find that the probable range of post-impact speed for the sport utility vehicle ranges from approximately 3.4 to 5.8 miles per hour, with a one-half foot difference in displacement distance of the vehicle. When we substitute this range of post-impact speeds back into **Equation 9**, we see that the resultant impact speed changes from 26.2 miles per hour to 44.8 miles per hour. The wide range of speeds found by making such a small change in the post-impact distance alone indicates the potential for error is too great for a reasonable conclusion to be reached. When one performs an uncertainty analysis with the probable ranges of the other variables of the calculation in this same example, we find that the resultant speed estimates are virtually useless.

The appropriateness of the methodologies discussed in this paper is dependent on the objectives of the collision analysis and individual factors of the collision itself. One should not make a blanket statement that momentum should not be used in motorcycle collision analysis because of potential errors in the speed estimates when there are numerous circumstances where these calculations prove to be extremely useful to the investigator. As with any tool in the accident reconstructionist’s toolbox, momentum should be used where it can be shown to be appropriate, but cannot be used in all circumstances.

Through the use of complementary methodologies, such as using both linear and rotational momentum on a collision, the investigator can travel different paths to arrive at the same destination. The use of multiple methodologies in motorcycle speed estimates and further evaluation of each of the methodologies with a sensitivity analysis will determine if the individual methods are suitable for the particular collision.

Summary and Conclusions

Historical publications by various authors have offered conflicting views on the appropriateness of conservation of linear momentum calculations in motorcycle collision analysis. While these works have appropriately warned of the potential limitations involved with these types of calculations, some simply dismiss the techniques without adequate explanation of when it should be dismissed and when it might be useful. The applicability of the momentum techniques discussed in this paper has been demonstrated to be appropriate under certain circumstances in motorcycle collision analysis. By properly evaluating the results of the momentum analysis with a sensitivity analysis, the reconstructionist can evaluate the applicability of these techniques to each individual collision.

As with all of the techniques available to the reconstructionist, there is no one single technique or methodology that can be applied in all circumstances. The proper use of linear and angular momentum techniques, as described in this paper, gives the reconstructionist other tools with which to evaluate motorcycle vs. automobile collisions.

References

1. Fricke, Lynn B., and Riley, Warner W., ***"Reconstruction of Motorcycle Traffic Accidents,"*** Topic 874 of the **Traffic Accident Investigation Manual**, Northwestern University Traffic Institute, 1990

2. Brown, John F., and Obenski, Kenneth S., **Forensic Engineering Reconstruction of Accidents,** Charles C. Thomas, 1990

3. Obenski, Kenneth S., **Motorcycle Accident Reconstruction: Understanding Motorcycles,** Lawyers & Judges Publishing Co., 1994

4. Niederer, Peter F., ***"Some Aspects of Motorcycle-Vehicle Collision Reconstruction,"*** SAE Paper 900750, Society of Automotive Engineers, 1990

5. Hurt, H. H., Jr.; Ouellet, J. V.; Thom, D. R.; ***"Motorcycle Accident Cause Factors and Identification of Countermeasures,"*** Traffic Safety Center, University of Southern California, 1981

6. Daily, John; Shigemura, Nathan S.; **Fundamentals of Applied Physics for Traffic Accident Investigators**, Institute of Police Technology and Management, 1997

7. MacInnis, Duane D., Cliff, William E., Ising, Kurt W., ***"A Comparison of Moment of Inertia Estimation Techniques for Vehicle Dynamics Simulation,"*** SAE Paper 970951, Society of Automotive Engineers, 1997

8. Garrott, W. Riley, ***"Measured Vehicle Inertial Parameters –NHTSA's Data Through September 1992,"*** SAE Paper 930897, Society of Automotive Engineers, 1993

9. Bartlett, Wade; Wright, William; Masonry, Oren; Brach, Raymond; Baxter, Al; Schmidt, Bruno; Navin, Frank; Stanard, Terry; ***"Evaluating the Uncertainty in Various Measurement Tasks Common to Accident Reconstruction,"*** SAE Paper 2002-01-0546, Society of Automotive Engineers, 2002

10. Brach, Raymond, ***"Uncertainty in Accident Reconstruction Calculations,"*** SAE Paper 940722, Society of Automotive Engineers, 1994

11. Kost, Garrison and Werner, Stephen M., ***"Use of Monte Carlo Simulation Techniques in Accident Reconstruction,"*** SAE Paper 940719, Society of Automotive Engineers, 1994

12. Tubergen, Renard G., ***"The Techniques of Uncertainty Analysis as Applied to the Momentum Equation for Accident Reconstruction,"*** SAE Paper 950135, Society of Automotive Engineers, 1995

13. Metz, L.D. and Metz, L.G., ***"Sensitivity of Accident Reconstruction Calculations,"*** SAE Paper 980375, Society of Automotive Engineers, 1998

14. Taylor, Barry N. and Kuyatt, Chris E., ***"Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results,"*** NIST Technical Note 1297, National Institute of Standards and Technology, 1994

15. Searle, John A. and Searle, Angela, ***"The Trajectories of Pedestrians, Motorcycles, Motorcylists, etc., Following a Road Accident,"*** SAE Paper 831622, Society of Automotive Engineers, 1983

16. Searle, John A., ***"The Physics of Throw Distance in Accident Reconstruction,"*** SAE Paper 930659, 1993

17. Severy, Derwyn M., Brink, Harrison M., Blaisdell, David M., ***"Motorcycle Collision Experiments,"*** SAE Paper 700897, Society of Automotive Engineers, 1970

18. Adamson, Kelley S.; Alexander, Peter; Robinson, Ed L.; Burkhead, Claude I.; McMannus, John; Anderson, Gregory C.; Aronberg, Ralph; Kinney, J. Rolley; Sallmann, David W.; Johnson, Gary M.; ***"Seventeen Motorcycle Crash Tests Held at WREX2000,"*** WREX2000 Conference Results, 2002

19. ***"Summary of Motorcycle Crash Test Results,"*** Joint Annual Conference of NATARI, MATAI, NAPARS, NJAAR AND NYSTARS, 1998

20. Motorcycle crash test results of the **20th Annual Special Problems in Traffic Crash Reconstruction**, 2002

Appendix A

The ***Rotational Analysis*** section of this paper explains the methodology used if the struck vehicle slides sideways instead of rotating about its center of mass. In very high speed collisions, the struck vehicle may rotate about its center of mass. In these circumstances the following analysis allows the investigator to estimate the impact speed for the motorcycle.

The first step is to determine the torque acting to slow vehicle's rotation as it travels to final rest. This can be accomplished through the following formula if we assume that the rotation of the vehicle is about its center of mass.

Equation A		
Formula	$\tau_{tire} = 2 \cdot WB \cdot f \cdot \left(\frac{W_f \cdot W_r}{W_t} \right)$	
Where	**W_f**	Weight on the front axle
	W_r	Weight on the rear axle
	W_t	Total weight of vehicle
	f	Coefficient of friction
	WB	Wheelbase of vehicle

The second step in the rotational analysis is to determine the angular velocity of the automobile after impact. This is accomplished through the Work-Energy Theorem in the following manner:

Equation B		
Formula	$\tau_{tire}\theta = \frac{1}{2}\mathrm{I}\omega^2$	
Where	**τ_{tire}**	Torque from sliding tires
	θ	Angular displacement in radians
	ω	Angular velocity in radians/second
	I	Yaw moment of inertia in lb-ft-sec^2

By solving **Equation B** for ***ω*,** we find the following to be true:

Equation C	
Formula	$\omega = \sqrt{\frac{2 \cdot \tau_{tire} \cdot \theta}{\mathrm{I}}}$

The next step in the analysis is to determine the amount of torque applied to the automobile during the collision to produce the calculated angular velocity. This is accomplished through Newton's Second Law for Rotation, where the following is true:

Equation D		
Formula	$\tau_{imp}\Delta t = \mathrm{I}(\omega_f - \omega_o)$	
Where	τ_{imp}	Torque from impact
	ω_f	Angular velocity of vehicle post-impact
	ω_o	Angular velocity of vehicle pre-impact
	Δt	Duration of collision impulse in seconds

or

Equation E	
Formula	$\tau_{imp} = \frac{\mathrm{I}(\omega_f - \omega_o)}{\Delta t}$

By using **Equation E** and assuming a time over which the collision impulse occurred, typically found to be within the range of .10 to .14 seconds, we can calculate the amount of torque acting to cause the vehicle to rotate.[14] Using the calculated range of torque acting on the vehicle to produce the rotation, we can determine the amount of force acting on the vehicle by dividing the torque by the length of the moment arm. The length of moment arm is determined by measuring the perpendicular distance from the principal direction of force (PDOF) to the center of mass of the vehicle.

Equation F		
Formula	$F = \frac{\tau_{imp}}{L}$	
Where	L	Length of moment arm

The next step in the rotational analysis is to determine the translational change in velocity experienced by the motorcycle as a result of the collision. Through Newton's Third Law of Motion, we know that the force acting on the automobile calculated in Equation (10) is the same magnitude as the force acting on the motorcycle. Using the ***Impulse = Momentum*** relationship, we can calculate the change in velocity experienced by the motorcycle in the following manner:

Equation G	
Formula	$F\Delta t = M\Delta V$

or

Equation H	
Formula	$\Delta V_m = \dfrac{F\Delta t}{M_m}$

This entire process can be combined into a single formula, which is derived by combining **Equations E**, **F**, and **H** to arrive at the following:

Equation I	
Formula	$\Delta V_m = \dfrac{I \cdot \omega}{L \cdot M_m}$

As one can see, **Equation I** gives a solution for the change in velocity experienced by the motorcycle using the variables for moment of inertia of the vehicle, the angular velocity of the vehicle, the length of the moment arm and the mass of the motorcycle. By using this single equation, we eliminate the need to estimate the time duration of the collision impulse and significantly simplify the methodology.

The initial speed of the motorcycle can then be estimated by using **Equation 8**, found in the ***Rotational Analysis*** section and derived in **Appendix B**.

Appendix B

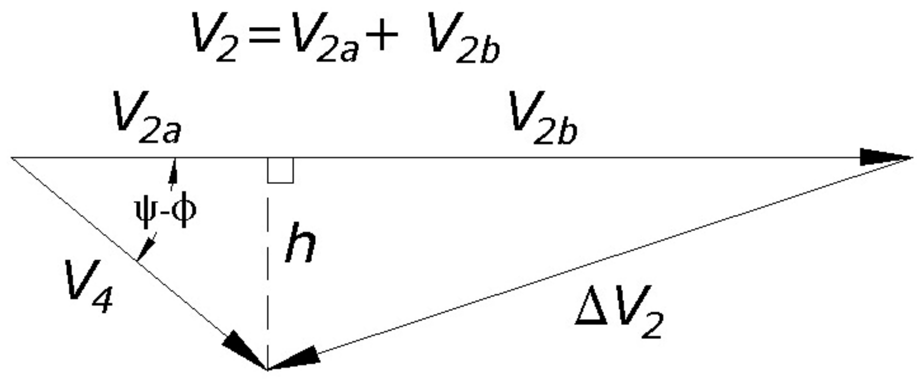


Figure 7 – Vector Diagram

From **Figure 7**, we can see that $\boldsymbol{V_2 = V_{2a} + V_{2b}}$. By breaking the vector diagram into two right triangles with ***h*,** we can solve for V_2 in the following manner.

Solving for V_2	
Step 1	$V_{2b} = \sqrt{\Delta V_m^2 - h^2}$
Step 2	$h = V_4 \cdot Sin(\psi - \phi)$
Step 3	$V_{2b} = \sqrt{\Delta V_m^2 - (V_4 \cdot Sin(\psi - \phi))^2}$
Step 4	$V_{2a} = V_4 \cdot Cos(\psi - \phi)$
Solution	$V_2 = V_4 \cdot Cos(\psi - \phi) + \sqrt{\Delta V_m^2 - (V_4 \cdot Sin(\psi - \phi))^2}$

Appendix C

Sensitivity Analysis 1 Numbers in bold are calculated. All other numbers are variables.							
	Weight of Car and Driver (lbs)	Weight of MC and Rider (lbs)	Speed of Car After Impact (mph)	Departure Angle of Car and MC (deg)	Approach Angle of Motorcycle (deg)	**V_2 (mph)**	**V_1 (mph)**
Est	2800.00	755.00	20.00	75.00	100.00	**92.37**	**10.90**
Min	2700.00	735.00	18.00	72.00	97.00		
Max	2900.00	780.00	22.00	78.00	103.00		
	Values used below				*95% Confidence*	**9.96**	**2.11**
					68% Confidence	**4.98**	**1.06**
	Min value used	Max value used	Est value used		*High Value*	**101.60**	**12.23**
					Low Value	**83.13**	**9.58**
	2700.00	755.00	20.00	75.00	100.00	**89.77**	**10.98**
	2800.00	780.00	20.00	75.00	100.00	**90.03**	**10.97**
	2800.00	755.00	22.00	75.00	100.00	**101.60**	**11.99**
	2800.00	755.00	20.00	78.00	100.00	**93.54**	**9.66**
	2800.00	755.00	20.00	75.00	103.00	**93.36**	**12.23**
	2900.00	755.00	20.00	75.00	100.00	**94.96**	**10.82**
	2800.00	735.00	20.00	75.00	100.00	**94.35**	**10.84**
	2800.00	755.00	18.00	75.00	100.00	**83.13**	**9.81**
	2800.00	755.00	20.00	72.00	100.00	**90.94**	**12.11**
	2800.00	755.00	20.00	75.00	97.00	**91.65**	**9.58**

Appendix D

Sensitivity Analysis 2 Numbers in bold are calculated. All other numbers are variables.

	Vehicle Depart. Speed (mph)	MC Depart. Speed (mph)	Rider Depart. Speed (mph)	MC Approach Angle (deg)	Vehicle Depart. Angle (deg)	MC Depart. Angle (deg)	Rider Depart. Angle (deg)	Vehicle Weight (lbs)	MC Weight (lbs)	Rider Weight (lbs)	**V_2 (mph)**	**V_1 (mph)**
Est	12.00	19.00	38.00	120.00	36.00	108.00	115.00	2770	618	190	**53.23**	**15.06**
Min	10.50	17.00	35.00	117.00	33.00	105.00	112.00	2700	600	180		
Max	13.50	21.00	41.00	123.00	39.00	111.00	118.00	2840	640	200		

Values used below		
Min value used	Max value used	Est value used

	V_2 (mph)	V_1 (mph)
95% Confidence	**4.85**	**1.99**
68% Confidence	**2.42**	**1.00**
High Value	**56.72**	**16.78**
Low Value	**49.74**	**13.34**

Vehicle Depart. Speed (mph)	MC Depart. Speed (mph)	Rider Depart. Speed (mph)	MC Approach Angle (deg)	Vehicle Depart. Angle (deg)	MC Depart. Angle (deg)	Rider Depart. Angle (deg)	Vehicle Weight (lbs)	MC Weight (lbs)	Rider Weight (lbs)	V_2 (mph)	V_1 (mph)
13.50	19.00	38.00	120.00	36.00	108.00	115.00	2770	618	190	**56.72**	**16.78**
12.00	21.00	38.00	120.00	36.00	108.00	115.00	2770	618	190	**54.91**	**15.17**
12.00	19.00	41.00	120.00	36.00	108.00	115.00	2770	618	190	**53.97**	**15.08**
12.00	19.00	38.00	123.00	36.00	108.00	115.00	2770	618	190	**54.97**	**16.03**
12.00	19.00	38.00	120.00	39.00	108.00	115.00	2770	618	190	**55.20**	**14.97**
12.00	19.00	38.00	120.00	36.00	111.00	115.00	2770	618	190	**52.94**	**14.81**
12.00	19.00	38.00	120.00	36.00	108.00	118.00	2770	618	190	**52.99**	**14.90**
12.00	19.00	38.00	120.00	36.00	108.00	115.00	2840	618	190	**53.94**	**15.03**
12.00	19.00	38.00	120.00	36.00	108.00	115.00	2770	640	190	**52.37**	**15.10**
12.00	19.00	38.00	120.00	36.00	108.00	115.00	2770	618	200	**53.07**	**15.07**
10.50	19.00	38.00	120.00	36.00	108.00	115.00	2770	618	190	**49.74**	**13.34**
12.00	17.00	38.00	120.00	36.00	108.00	115.00	2770	618	190	**51.55**	**14.95**
12.00	19.00	35.00	120.00	36.00	108.00	115.00	2770	618	190	**52.49**	**15.04**
12.00	19.00	38.00	117.00	36.00	108.00	115.00	2770	618	190	**51.74**	**14.15**
12.00	19.00	38.00	120.00	33.00	108.00	115.00	2770	618	190	**51.18**	**15.12**
12.00	19.00	38.00	120.00	36.00	105.00	115.00	2770	618	190	**53.48**	**15.31**
12.00	19.00	38.00	120.00	36.00	108.00	112.00	2770	618	190	**53.45**	**15.22**
12.00	19.00	38.00	120.00	36.00	108.00	115.00	2700	618	190	**52.53**	**15.09**
12.00	19.00	38.00	120.00	36.00	108.00	115.00	2770	600	190	**53.97**	**15.03**
12.00	19.00	38.00	120.00	36.00	108.00	115.00	2770	618	180	**53.40**	**15.05**

Appendix E

Sensitivity Analysis 3 Numbers in bold are calculated. All other numbers are variables.

	Wheelbase V_1 (feet)	Weight on axle closest to impact (lbs)	Friction (f)	Angle of Rotation (degrees)	Moment of Inertia	Weight of vehicle (lbs)	Weight of Motorcycle (lbs)	Length of Moment Arm (feet)	Change in Angle of Motorcycle (degrees)	Post-impact speed of Motorcycle (MPH)	**Torque - Tire (lb-ft)**	**Angular Velocity (rad/sec)**	**Change in Velocity (fps)**	**Change In Velocity (mph)**	**Impact Speed of MC (mph)**
Est	8.6	1025	0.75	145	1617.3	2770	683	8.5	12	19	**6611.25**	**3.67**	**50.61**	**34.52**	**52.88**
Min	8.5	1000	0.7	142	1540	2700	620	8	9	17					
Max	8.7	1050	0.8	148	1700	2840	808	9.00	15.00	21					

	Torque - Tire (lb-ft)	Angular Velocity (rad/sec)	Change in Velocity (fps)	Change In Velocity (mph)	Impact Speed of MC (mph)
95% Confidence	**475.58**	**0.14**	**7.54**	**5.14**	**5.53**
68% Confidence	**237.79**	**0.07**	**3.77**	**2.57**	**2.77**
High Value	**7052.00**	**3.79**	**55.76**	**38.03**	**56.41**
Low Value	**6170.50**	**3.54**	**42.78**	**29.18**	**47.50**

Values used below		
Min value used	Max value used	Est value used

Wheelbase V_1 (feet)	Weight on axle closest to impact (lbs)	Friction (f)	Angle of Rotation (degrees)	Moment of Inertia	Weight of vehicle (lbs)	Weight of Motorcycle (lbs)	Length of Moment Arm (feet)	Change in Angle of Motorcycle (degrees)	Post-impact speed of Motorcycle (MPH)	**Torque - Tire (lb-ft)**	**Angular Velocity (rad/sec)**	**Change in Velocity (fps)**	**Change In Velocity (mph)**	**Impact Speed of MC (mph)**
8.70	1025.00	0.75	145.00	1617.30	2770.00	683.00	8.5	12	19	**6688.13**	**3.67**	**51.11**	**34.87**	**53.23**
8.60	1050.00	0.75	145.00	1617.30	2770.00	683.00	8.5	12	19	**6772.50**	**3.68**	**51.67**	**35.24**	**53.61**
8.60	1025.00	0.80	145.00	1617.30	2770.00	683.00	8.5	12	19	**7052.00**	**3.79**	**52.27**	**35.66**	**54.02**
8.60	1025.00	0.75	148.00	1617.30	2770.00	683.00	8.5	12	19	**6611.25**	**3.70**	**51.13**	**34.88**	**53.24**
8.60	1025.00	0.75	145.00	1700.00	2770.00	683.00	8.5	12	19	**6611.25**	**3.61**	**51.45**	**35.09**	**53.46**
8.60	1025.00	0.75	145.00	1617.30	2840.00	683.00	8.5	12	19	**6611.25**	**3.68**	**50.39**	**34.38**	**52.73**
8.60	1025.00	0.75	145.00	1617.30	2770.00	808.00	8.5	12	19	**6611.25**	**3.67**	**42.78**	**29.18**	**47.50**
8.60	1025.00	0.75	145.00	1617.30	2770.00	683.00	9	12	19	**6611.25**	**3.67**	**47.80**	**32.61**	**50.95**
8.60	1025.00	0.75	145.00	1617.30	2770.00	683.00	8.5	15	19	**6611.25**	**3.67**	**50.61**	**34.52**	**52.53**
8.60	1025.00	0.75	145.00	1617.30	2770.00	683.00	8.5	12	21	**6611.25**	**3.67**	**50.61**	**34.52**	**54.79**
8.50	1025.00	0.75	145.00	1617.30	2770.00	683.00	8.5	12	19	**6534.38**	**3.66**	**50.11**	**34.18**	**52.54**
8.60	1000.00	0.75	145.00	1617.30	2770.00	683.00	8.5	12	19	**6450.00**	**3.65**	**49.57**	**33.81**	**52.17**
8.60	1025.00	0.70	145.00	1617.30	2770.00	683.00	8.5	12	19	**6170.50**	**3.54**	**48.90**	**33.35**	**51.70**
8.60	1025.00	0.75	142.00	1617.30	2770.00	683.00	8.5	12	19	**6611.25**	**3.63**	**50.09**	**34.17**	**52.52**
8.60	1025.00	0.75	145.00	1540.00	2770.00	683.00	8.5	12	19	**6611.25**	**3.73**	**49.82**	**33.98**	**52.34**
8.60	1025.00	0.75	145.00	1617.30	2700.00	683.00	8.5	12	19	**6611.25**	**3.65**	**50.84**	**34.68**	**53.04**
8.60	1025.00	0.75	145.00	1617.30	2770.00	620.00	8.5	12	19	**6611.25**	**3.67**	**55.76**	**38.03**	**56.41**
8.60	1025.00	0.75	145.00	1617.30	2770.00	683.00	8	12	19	**6611.25**	**3.67**	**53.78**	**36.68**	**55.05**
8.60	1025.00	0.75	145.00	1617.30	2770.00	683.00	8.5	9	19	**6611.25**	**3.67**	**50.61**	**34.52**	**53.16**
8.60	1025.00	0.75	145.00	1617.30	2770.00	683.00	8.5	12	17	**6611.25**	**3.67**	**50.61**	**34.52**	**50.97**